eccentricity.[7] As shown in the schematic in Fig. 3, this occurs if the deployment rate has a Fourier component equal to the orbital period.

Energy Storage

Note that the tether need not be an otherwise inactive part of the satellite. Since energy is gained in deploying the tether out and used in pulling the tether in, it could be used for energy storage. An optimal design for a geosynchronous satellite might well use this energy storage in place of batteries to power the satellite electrical systems during the eclipse period (which lasts about 1.1 h per day for a period of several days around the equinox) when no solar energy is available to power the solar cells. The small orbital change produced by this could be easily corrected by reeling the tether in to compensate during the sunlit portion of the orbit. By integrating Eq. (1), the total energy available is

$$E = 1.5 m g L^2 r_e^2 / r_o^3 \tag{8}$$

For E, the energy in W-h, L, the half length in km, and m, the satellite mass in kg, in GEO the specific energy is 0.55 W-h/kg of satellite mass. From Eq. (2), the maximum energy that can be stored per kilogram tether mass is

$$E/m_t = 0.5\, gL_c \tag{9}$$

independent of orbital radius. For a Kevlar cable, this is about 350 W-h/kg. Even including a large safety margin, this compares favorably to the 14 W-h/kg produced by current technology Ni-H batteries.

The tether may have additional uses on the satellite as well. In addition to correction of small perturbations in the orbital period (e.g., due to lunar and solar pertubations) by adjusting the tether length, modulation of the length in phase with the orbital period could be used to adjust changes in the orbital eccentricity.[7,8] Another possible use for the tether is as an element of the initial boost of the satellite into position by the well-researched process of tether boost.[1,2] Finally, the requirement to remove the satellite from geosynchronous orbit at the end of life can be accomplished by cutting the tether at full extension.

Conclusions

A new method has been described for a satellite in orbit to be repositioned in orbital longitude by use of a tether connecting the active satellite with an inert mass, such as the expended booster. In addition to allowing repositioning, the system also allows correction of small changes in orbital period and eccentricity and use of the tether as an energy storage source.

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Attitude and Spin Rate Control of a Spinning Satellite Using Geomagnetic Field

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Introduction

THE use of magnetic control torques resulting from the interaction between the geomagnetic field and a coil magnetic moment is one of the earliest satellite attitude control methods.[4-6] This Note studies both the spin axis reorientation as well as the spin rate control of the Brazilian Data Collecting Satellite (BDCS). The solution of the spin axis reorientation problem is based on the work of Shigehara.[3] The spin rate control is performed by the appropriate switching of a plane magnetic coil.

Dynamic Modeling

The differential equations for the attitude motion of spin-stabilized satellites were developed using the spin axis spherical coordinates α (right ascension) and δ (declination) and spin rate ω. From Ref. 1, we have

$$I_z \omega \cos\delta \dot{\alpha} \boldsymbol{I} + I_z \omega \dot{\delta} \boldsymbol{J} + I_z \dot{\omega} \boldsymbol{K} = \boldsymbol{T}_1 \tag{1}$$

where I_z is the moment of inertia about the z axis (spin axis); $\boldsymbol{I}$, $\boldsymbol{J}$, and $\boldsymbol{K}$ are the unit vectors in the inertial coordinate system, and $\boldsymbol{T}_1$ is the resulting torque acting on the satellite. It is given by $\boldsymbol{T}_1 = \boldsymbol{T}_a + \boldsymbol{T}_p + \boldsymbol{T}_e$ where $\boldsymbol{T}_a$ and $\boldsymbol{T}_p$ are the torques representing the interaction between the geomagnetic field and magnetic moments along and orthogonal to the spin axis, respectively; $\boldsymbol{T}_e$ is the torque arising from the eddy currents[1]:

$$\boldsymbol{T}_a = -(m_i + m_r)(B_y \boldsymbol{i} - B_x \boldsymbol{j}) \tag{2a}$$

$$\boldsymbol{T}_p = v\,[m_p \sin\psi\, B_z \boldsymbol{i} - m_p \cos\psi\, \boldsymbol{j} + (m_p \cos\psi\, B_y - m_p \sin\psi\, B_x)\, \boldsymbol{k}] \tag{2b}$$

$$\boldsymbol{T}_e = p\,\omega\,[B_x B_z \boldsymbol{i} - B_y B_z \boldsymbol{j} + (B_x^2 + B_y^2)\, \boldsymbol{k}] \tag{2c}$$

In the foregoing expressions, m_r is the residual magnetic moment and m_i and m_p are the magnetic moments generated by the axis and plane coil, respectively; B_x, B_y, and B_z are the components of the geomagnetic field[2] in the satellite coordinate system ($\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$), $\boldsymbol{k}$ being aligned with the spin axis; $v \in (-1; 0; +1)$ is the polarity of the plane coil; ψ is the phase angle of the plane coil, and p is a constant depending on the satellite geometry and material conductivity.[1]

Control Law

Spin Axis Attitude Control

The spin axis can be steered between two given attitudes by conveniently switching the axis magnetic coil. The control

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policy to be derived follows closely the work of Shigehara.[3] It is assumed that the satellite angular momentum is parallel to the spin axis as a consequence of the action of a nutation damper. Denoting by $\boldsymbol{k}_f$ the unit vector corresponding to the desired attitude of the spin axis and by $\boldsymbol{H}$ the actual angular momentum, we can define an error vector $\boldsymbol{E}$ as:

$$\boldsymbol{E} = \boldsymbol{k}_f - \frac{\boldsymbol{H}}{H} \tag{3}$$

where $H = |\boldsymbol{H}|$. The objective of the control action is thus to reduce $\boldsymbol{E}$ to zero. Differentiating Eq. (3) with respect to time and taking into account that $\boldsymbol{T}_m = \dot{\boldsymbol{H}}$, we obtain

$$\dot{\boldsymbol{E}} = \left(\frac{-1}{H}\right)\boldsymbol{T}_m + \left(\frac{\dot{H}}{H^2}\right)\boldsymbol{H} \tag{4}$$

Considering that $\boldsymbol{T}_m$ is orthogonal to $\boldsymbol{H}$ and after some algebraic manipulation we get $\dot{H} = 0$. Hence, if we take into account Eq. (4), it follows that

$$\frac{\mathrm{d}(E^2)}{\mathrm{d}t} = \left(\frac{-2}{H}\right)\boldsymbol{E}\cdot\boldsymbol{T}_m \tag{5}$$

Recalling that the torque corresponding to the axis coil can be expressed as

$$\boldsymbol{T}_m = um_i\,\boldsymbol{k}\times\boldsymbol{B} \tag{6}$$

where $u\in(-1;\ 0;\ +1)$ is the polarity of the axis coil and $\boldsymbol{B} = B_x\boldsymbol{i} + B_y\boldsymbol{j} + B_z\boldsymbol{k}$, we can rewrite Eq. (5) as

$$\frac{\mathrm{d}(E^2)}{\mathrm{d}t} = \left(\frac{-2m_i}{H}\right)u\cdot s_1 \tag{7}$$

where s_1 is defined as $s_1 = \boldsymbol{E}\cdot(\boldsymbol{k}\times\boldsymbol{B})$.

Notice that s_1 can be viewed as a switching function since it determines at each time the value of u appropriate to decrease the attitude error. In other words, $u = +1$ if $s_1 > 0$; $u = 0$ if $s_1 = 0$; $u = -1$ if $s_1 < 0$.

Spin Rate Control

Spin rate control is performed using a plane magnetic coil whose axis is orthogonal to the spin axis. To increase the satellite spin rate, the torque $\boldsymbol{T}_p$ must be such that $\boldsymbol{T}_p\cdot\boldsymbol{k} > 0$, which, in view of Eq. (5), can be rewritten as

$$vm_p\,(\cos\psi\,B_y - \sin\psi\,B_x) > 0 \tag{8}$$

If we define the switching function s_2 as $m_p(\cos\psi\,B_y - \sin\psi\,B_x)$, the plane magnetic coil polarity is then chosen as $v = +1$ if $s_2 > 0$; $v = 0$ if $s_2 = 0$; $v = -1$ if $s_2 < 0$.

Control variable v is taken as zero whenever the difference between the actual and nominal spin rates is smaller in magnitude than a fixed threshold.

Results

A simulation program has been developed to test the attitude and spin rate control laws. Both attitude stabilization and reorientation of the spin axis are demonstrated in the example

Table 1 Sequence of attitudes

Attitude	α, deg	δ, deg
$A0$	127	20
$A1$	120	89
$A2$	30	70
$A3$	20	−30
$A4$	10	−70
$A5$	0	−89

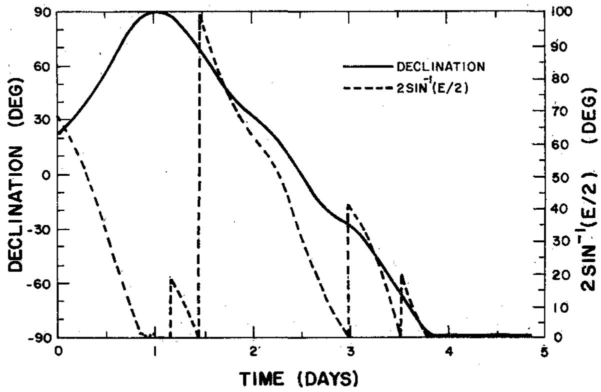

Fig. 1 Declination and error angle vs time.

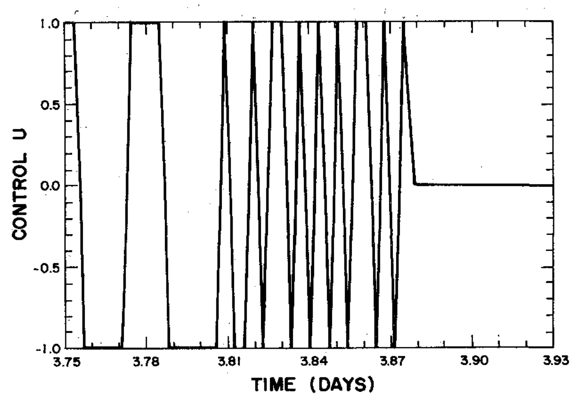

Fig. 2 Control u vs time.

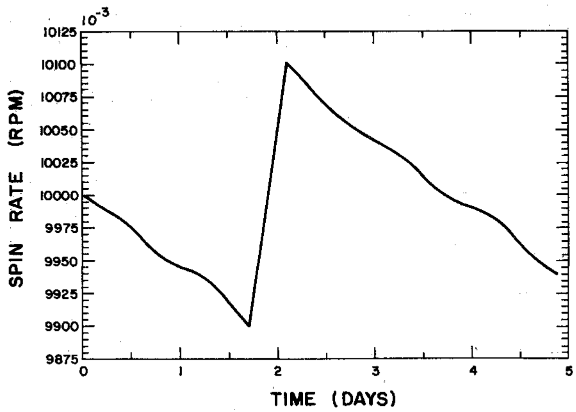

Fig. 3 Spin rate vs time.

that follows. In the case of attitude stabilization, the goal is to keep the satellite attitude constant, whereas in the case of attitude reorientation, the objective is to maneuver the satellite between its actual and a desired attitude. In specific problems, it can happen that a given set of attitudes is not allowed during the reorientation maneuver. In these cases, the user can define an intermediate sequence of attitudes in such a way as to not violate the aforementioned constraints. The simulation is then carried out between successive attitudes until the final desired orientation is attained within a prescribed accuracy.

The example of a reorientation maneuver is now discussed. For the sake of easy referencing in what follows, the attitudes of interest are provided in Table 1.

The satellite initial attitude is $A0$. The first phase of the maneuver corresponds to the initial acquisition of the spin axis (attitude $A1$). Once attitude $A1$ is reached, it is stabilized during about four orbits. Then the satellite is tumbled (attitude $A5$). This part of the maneuver is performed by attaining the sequence of intermediate attitudes $A2$–$A4$. Finally, the spin axis is stabilized at attitude $A5$ until the end of the simulation. The following parameters were assumed in the simulation: $m_r = -0.1 \text{Am}^2$, $m_i = 15\text{Am}^2$, $m_p = 0.5\text{Am}^2$, $p = 1925.89$ m^4/Ω, $I_z = 10$ kgm^2, $I_x = 9.8$ kgm^2, altitude 750 km (circular orbit), inclination 25 deg.

Figure 1 illustrates both the satellite attitude behavior and error angle $2 \sin^{-1}(E/2)$ during the reorientation maneuver. Figure 2 is a plot of the control variable u as a function of time during approximately three orbits. The effect of the spin rate control is shown in Fig. 3. The nominal spin rate has been taken as 10 rpm and the maximum error allowable is 0.1 rpm. Since the attitude dynamics is slow with respect to the spin period, the mean value of the plane magnetic coil torque in one period has been used in the integration of $\dot{\omega}$ [see Eq. (1)].

Conclusions

An example of the practical application of the ideas contained in Ref. 3 to the reorientation of the spin axis as well as the spin rate control of the BDCS has been presented. The simulation results suggest the feasibility of performing the reorientation maneuvers by ground command; furthermore, the spin rate control can be done by an onboard computer using magnetometer phase signals to properly switch the plane coil.

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Approach to Robust Control Systems Design

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Introduction

THE problem of robustness has received considerable attention in recent years. Some of the recent research in this area was inspired by the Kharitonov theorem related to the asymptotic stability of a family of systems described by linear differential equations.[1] Different generalizations of the Kharitonov theorem have been discussed in Refs. 2 and 3.

Based on the criteria similar to the well-known ones in the classical automatic control theory, the problem of robust stability in linear time-invariant closed-loop systems with parametric uncertainty is formulated mostly, in essence, as the analysis problem. Using classical techniques, the stability of the system under consideration can be examined and the controller parameters, which guarantee the asymptotic stability of the closed-loop system, can be determined. Usually, it is assumed that a controller has been constructed for the so-called nominal plant. By using the Kharitonov-type theorem, one can only check if the system with perturbed parameters remains stable. No effective design procedure exists for changing the controller parameters to make the real (not nominal) system asymptotically stable.

Parallel with the approach just discussed, the Lyapunov-Bellmen method is used to design controllers for a system with uncertain dynamics (see, e.g., Refs. 4 and 5).

This paper presents an approach to the synthesis of robust control systems with uncertain parameters which is based on the consideration of an optimal control problem with a specified performance index. The introduced estimate of the location of the eigenvalues of the plant with uncertain parameters allows us to formulate the optimal control problem, the solution of which guarantees the asymptotic stability of the closed-loop system. The main advantage of the proposed procedure is its simplicity. It is similar to the well-known procedure of the analytical controller design based on the solution of the linear quadratic optimal control problem with the integral quadratic performance index.

Statement of the Problem and Main Results

Let us consider the linear controllable plant described by the equation

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

where $\boldsymbol{x}$ is an n-dimensional state-space vector, $\boldsymbol{u}$ is an m-dimensional control vector, and $A = |a_{ij}|$ and $B = |b_{ij}|$ are matrices of appropriate dimensions.

It is assumed first that only elements of the state matrix A are not known exactly, i.e.,

$$A_1 \leq A \leq A_2 \tag{2}$$

where $A_1 = |a_{1ij}|$ and $A_2 = |a_{2ij}|$ characterize the upper and lower bounds of A, respectively.

The robust control problem consists of finding controller equations that make the closed-loop system asymptotically stable for all state matrices of the form of Eq. (2).

The well-known procedure of analytical controller design for systems of the form of Eq. (1) is based on minimization of the functional

$$J_0 = \frac{1}{2} \int_0^\infty [x^T(t)Qx(t) + cu^T(t)u(t)]\, \mathrm{d}t \tag{3}$$

where $Q = |q_{ij}|$ is a non-negative definite symmetric matrix, and c is a positive constant.

The control law has the form

$$u(t) = -\frac{1}{c} B^T W x(t) \tag{4}$$

where the positive-definite matrix W satisfies the Riccati equation.

Unlike the standard procedure of the analytical controller design, in the case under consideration we lack the proper information to calculate the matrix W.

It is known[6] that minimization of the functional

$$J_1 = \frac{1}{2} \int_0^\infty e^{2\gamma t}[x^T(t)Qx(t) + cu^T(t)u(t)]\, \mathrm{d}t \tag{5}$$

subject to the boundary conditions described by system (1) is equivalent to minimization of the functional (3) subject to the

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